Abstract: Minimum Variance Analysis (MVA) is frequently used for the geometrical reorganization of vectors whose properties are unknown. The Coplanarity Variance Analysis (CVA) developed in this paper reproduces the layer geometry involving coplanar magnetosonic shocks or plane polarized wave trains (including normals and coplanarity directions) 300 times more precisely (<0.1°) than MVA using the same input data. The CVA technique exploits the eigenvalue degeneracy in MVA at planar structures to enforce coplanarity in the search for the geometry. Together CVA and MVA may be used to sort between the hypotheses that the time series is a one dimensional layer that is either (i) coplanar (shocks/plane polarized waves), or (ii) torsional (rotational/tangential discontinuities), or (iii) neither (i) or (ii).

1. Introduction

Minimum Variance Analysis (MVA) was invented to survey time profiles of magnetometer data at the magnetopause, looking for rotational discontinuities (RD) in current layers (Sonnerup and Cahill, 1968). From a geometrical point of view RD’s and tangential discontinuities are vectors fields containing significant torsion. The MVA assumes that the RD is a planar structure; with Gauss’s law this assumption implies that the component of $\mathbf{B}$ normal to the phase front $B_n$ should be constant. The MVA problem is reducible to a 3x3 eigenvalue problem for a real symmetric matrix whose positive eigenvalue’s are the variances of the field along the principal axes recently reviewed by Sonnerup and Scheible, (1998). The minimum eigenvalue $\lambda_1$ determines a related eigenvector direction $\zeta_1$ which MVA identifies with the
normal \( \mathbf{n} \) to the current sheet. By theorem the eigenvector’s \( \zeta_j \) of the covariance matrix of MVA are orthogonal; they form a convenient basis to define a transformation from the original sensor coordinates to a “minimum variance” coordinate system. The ease of making such a transformation causes it to be used frequently.

This paper shows the deficiency of such an approach for finding the geometry of all one-dimensional current layers. Besides torsional RD/TD layers there are 1-D layers that intrinsically lack torsion, such as linearly polarized waves or shocks, collectively referred to below as Coplanar Wave Disturbances (CWD’s). This paper suggests an appropriate approach for CWD’s in the form of Coplanarity Variance Analysis (CVA). Even though it operates on the same input data as MVA, CVA very accurately (\( \ll 1^\circ \)) reconstructs the shock geometry (k-vector) and the orientation of the coplanarity (polarization) plane across a broad range of realistic slow and fast modeled shock profiles. Of course CVA does not replace full Rankine-Hugoniot (R-H) fitting procedures (e.g. Vinas and Scudder, 1986) but may be the “best” available source of geometry for CWD’s when, for example, the correlative measurements at a shock necessary to perform R-H tests are unavailable at the requisite time resolution.

Since the magnetic field \( \mathbf{B} \) in an ideal RD is comprised of a constant normal component with the field transverse to the normal executing a net rotation through \( 0 < \phi \leq 180 \), a coordinate system with one axis aligned with the normal direction will only record variance along the coordinate axes transverse to the normal. Identifying the direction of minimum variance of MVA with the normal to the 1-D current sheets with torsion (RD/TD) makes sense for this reason. It does not follow, however, that all 1-D current sheets will have their normals accurately found by MVA.
One dimensional CWD’s are supported by 1-D current layers where $B_n$ would be expected to be nearly constant. CWD layers must also satisfy the coplanarity conditions while the RD need not. The new Coplanarity Variance Analysis (CVA) technique proposed below reconstructs a coordinate system under two simultaneous assumptions: (i) that the average transformed value $B$ perpendicular to the coplanarity plane vanish (sufficiently well), while (ii) simultaneously minimizing the variance (sufficiently well) along the normal/wave vector. Because CVA requires more than does MVA, its geometry may or may not be preferred for any given time series to that suggested by MVA. As outlined below the tandem of CVA and MVA can be used to find 1-D coplanar layers, while sorting them from torsional layers such as RD’s and (TD’s), and also creating a third subset that is inconsistent with the 1-D premise and classification in either the torsional or planar subgroup. In this way MVA should be thought of as a means of testing the hypothesis, “Is this a torsional sheared one-dimensional layer?”, while CVA tests the hypothesis, “Is this a coplanar sheared one-dimensional layer?”. To facilitate its proper use, MVA should perhaps be called Torsional Variance Analysis, “TVA”.

2. Symptoms of the Problem

A time series of a torsional magnetic field in the form of an isomagnetic RD transition is illustrated in terms of the Cartesian components of $B$ by the solid curves of Figure 1A. The coordinates were chosen so that the initial magnetic field direction was in the x-z plane. The normal component is shown in light blue (cyan), while the transverse components are illustrated in red (y) and green (z) traces. The dashed curve of the corresponding color is the arithmetic mean of that Cartesian component. Colored flags anchored by a diamond reflect the standard deviation in that component about its mean. As depicted the steady normal component has a small variance. The other two components have variances that are larger owing to the coherent
jumps in the averages of both of these components. In fact it is the substantial variance in both transverse components that define the layer as possessing DC torsion. This situation clearly leaves the direction of minimum variance as a distinguishable direction if it can be found, possessing the least (ideally no) variance.

Figure 1B illustrates the same RD of Figure 1A in the standard “minimum variance” coordinates where the MVA eigenvectors’s have become the new basis directions: \( x' = \zeta_1; \)
\( y' = \zeta_2; \) and \( z' = \zeta_3 \). The standard deviations in the new coordinate system are also shown. The two Figures of Merit in the MVA panel are defined by \( \text{FOM}=Q_1, Q_2, \) where \( Q_j = \lambda_j/(\lambda_1 + \lambda_2 + \lambda_3) \). By theorem the sum of the variances cannot change under coordinate rotations. In the MVA coordinate system, however, the variance has been repartitioned among the transformed components of the magnetic field: increasing the variance of the \( z' \) components, while compensating by lowering the variance along the \( y' \) direction. Eigenvalues are coordinate invariant properties of the geometry. The ratio \( \lambda_2/\lambda_1 = Q_2/Q_1 \sim 8 \) in this example approaches the recommended (Sonnerup and Scheible, 1998) ratio of 10 for good geometry from RD’s. Since the variance is positive and it was already essentially 0 along \( x \), it is not surprising that the minimum variance direction, \( x' \), is selected parallel to \( x \) with essentially the same variance about it. (For the coordinate rotation between the two frames (Figure 1-A \( \rightarrow \) Figure 1-B) the sum of the eigenvalue’s for eigenvector’s transverse to the normal is preserved as a corollary to the invariance of the sum of eigenvalue’s and the null variance along the \( x \) coordinates in both initial and final frames considered here.)

The other propagating discontinuities in MHD are the fast, intermediate and slow shocks. A sub-set of the Rankine-Hugoniot jump conditions (beyond Gauss’s law) imply that the asymptotic parts of the field (away from the transition) satisfy the magnetic “Coplanarity”
Theorem. The same coplanar geometry of the fields is a property of any linearly polarized wave train. The term Coplanar Wave Disturbance (CWD) embraces both shocks and plane polarized wave trains. We show examples of candidate of each subtype of CWD in Figures 4 and 5 below. The magnetic fields on either side of the shock, as sketched in Figure 2, should have no average components perpendicular to a yet to be found coplanarity plane parallel to the two planes labeled C, C’ in Figure 2. This plane “contains” the shock normal \( \mathbf{n} \), the net vectorial changes of the magnetic field and the fluid velocity. The statements of “contain” and are “perpendicular” are in the sense of vector algebra of a vector field and MHD; these local geometrical statements do not imply the global geometrical properties of tubes of force that pierce the shock layer. For example, the tube of magnetic force cross the shock (S) layer migrates perpendicular to C and thus the coplanarity plane within the current carrying shock layer enroute to the geometrically equivalent parts of “the” coplanarity plane labeled C’; cf. Scudder, 1995, Fig 15. A plane polarized wave train would have the same coplanar geometry, but the field lines throughout the disturbance would lie in one geometrical plane. If the normal to the coplanarity plane is identified as the \( y'' \) direction in CVA, there is the requirement in this system that \( \langle B_y \rangle = 0 \), where the angular brackets imply the time average. In general there must be changes in a CWD in the remaining transverse direction we will call \( z'' \), so that there is current perpendicular to this plane either in the shock front (S) or throughout the wave field transverse to the wave vector. Like the component along the wave vector, the components of the CWD out of the coplanarity plane are ideally expected to be constant, and zero. By the same argument that motivated MVA as a minimization problem, the variance of the components out of the coplanarity plane should also be small for CWD.
Therefore to find a geometry suitable for CWD’s a coordinate system must be found where both $\sigma_x$ and $\sigma_y$ are small while $< B(t) > \cdot \mathbf{y}'' = 0$. The first two of these requirements explains how MVA can fail to obtain good CWD normals; a CWD possesses two (!) “minimum” variance directions, presenting two orthogonal directions of “essentially” the same very small variance. This is the statement of the symmetry that a CWD possesses that a torsional sheared layer does not have. While it is true that there may still be “a” computed minimum variance direction, the real question is whether its distinction from the intermediate eigenvalue’s eigenvector direction is meaningful. In the language of linear algebra the eigenvalue spectrum for CWD time series is (computationally) degenerate in the presence of coplanar symmetry in the vector field being surveyed. For shocks whose internal structure is sensed, and as Figure 2 illustrates, there is a small net average value of $< B(t) > \cdot \mathbf{y}''$; (Goodrich and Scudder, 1984, and Tidman and Krall, 1971) that will cause slight errors to be made on requiring the chosen CVA coordinate system to use. This average value is what remains after summing the non-zero contributions in the shock layer (S) with zero mean contributions outside of it. Depending on the fraction of data included in the time series containing the current layer, there may be a slight preference for a lower variance along the normal relative to the $\mathbf{y}''$ direction. Offsetting this surmise are 2-D effects that impact the constancy of the normal component and pre- and post-shock wave trains that contribute to both variances, leaving it a toss up which directions (normal or along coplanarity normal) that should have the lesser variance.

Panel D in Figure 1 depicts a fast shock magnetic field profile (as a concrete example of a CWD) in the spacecraft frame, using the same colors as the other panels for its x (cyan), y (red), and z (green) components. In this frame there is usually variance along all three Cartesian directions. Figure 1E illustrates the MVA reorganization attempt for the shock time series and
variances. The FOM values are comparable to one another, unlike the situation in Panel 1B where for the RD the FOM were distinct in the ratio exceeding 8. The ratio of eigenvalues is 2.7, well below the recommended value of 10 for confident geometry inversion. The minimum variance red (x’’) and cyan (y’’) traces have mean values both removed from zero.

However, panel F of Figure 1 illustrates the reorganization afforded by the CVA technique developed below. The new x’’ (red) trace is steady, while the cyan trace reflects \( B_y \). The z’’ component of \( B \) increases, and this is recognized as a potential shock candidate. CVA has found the “correct” underlying shock solution. The FOM for CVA is the standard deviation along the newly selected x’’ and y’’ directions divided by the invariant Pythagorean sum of standard deviations. The FOM are approximately equal, and more equal than the FOM’s for the MVA assay of the same time series in panel E. The separate totals of the FOM’s from MVA and CVA for the same time series are, by construction, equal; the CVA choice has enforced coplanarity at the price of slightly increasing the variance along the selected normal. The equipartition of the FOM at small values in CVA is noteworthy, since as we see here it confirms that the CVA organization is compatible with CWD, and an affirmation that the geometrical construction of CVA is more than linear algebra.

An interesting question remains with real data, “How does one tell whether one should stop at the MVA or the CVA geometry inversion?” A clue is in the FOM of the MVA technique when applied to different time series: Panels 1B (RD) and Panel 1E (Fast Shock). The FOM for MVA on the RD are distinct with ratio 8+, clearly well separated on the interval of \([0,1]\). By contrast the FOM for MVA on the shock layer have ration 2.7 in Figure 1E, while formally distinct, are operationally small and well below the ratio of 10 recommended for accurate geometry inversions at torsional layers. If the MVA situation on a new time series looks like the
FOM in Panel 1E, it is appropriate to press on with CVA analysis developed in this paper. The price of not pursuing this distinction can leave the physics of the layer unfocussed.

To shed further light on this problem we complete our graphical discussion of Figure 1 with Panel C, where CVA analysis has been blindly performed on a layer simulated as a RD. In seeking to enforce a shock like CVA solution with \( \langle B_{z'} \rangle = 0 \), this procedure has increased the variance in CVA along the suggested normal (cyan) trace FOM(1), giving to it nearly all the very substantial variance that used to be along the \( y' \) direction in MVA. By the nature of CVA to be discussed below, the \( z'' \) axis in CVA is the same physical direction as in MVA, so the particular “reconstruction” in Panel C selected by the algorithm has effectively mixed the \( x' \) and \( y' \) components of MVA enroute to satisfying the CVA algorithm. It is not so damaging that CVA \( Q_1 \) is preferentially increased by CVA in Panel 1c; rather it is that by performing CVA blindly on the time series that a large ratio of eigenvalues with MVA is inverted upside down with the implication that in CVA the variance along the normal is 8+ times the variance in the direction normal to the coplanarity plane. The clear indication that CVA is inappropriately applied in this time series is that \( Q_1 \) in CVA is comparable to \( Q_2 \) of MVA on the RD that we have already said was clearly distinguishable from, that is 8+ times larger than, \( Q_1 \). In a more positive vein, the fact that \( Q_1 \) is not substantially the same as \( Q_2 \) after applying CVA implies that the CVA geometry is unlikely to be correct. Conversely in Panel 1F, for the shock simulation, when the final result and correct geometry was acquired \( Q_1, Q_2 \) under CVA were essentially the same, even though the algorithm described below does not enforce this condition (as can also be seen in Panel 1C).

3. Variance Solution for the CWD Problem:
The eigenvector ζ³ associated with the maximum eigenvalue of the MVA’s covariance matrix is the most robust eigenvector for either the RD or shock layers even in the presence of 2-D effects and even when \( Q_2 / Q_1 \ll 10 \) (cf. statistical study summarizing over 100,000 MVA inversions: Scudder et al, 2004a). In CVA we identify \( \zeta_3 \) with the direction transverse to the normal (wave vector) and parallel to the components of \( B \) that participate in the net compression at the shock (undulations in the CWD). Such an eigenvector will be oriented in the phase front of a shock (S), transverse to the normal and in the coplanarity plane parallel to C, C’ of Figure 2. In the CVA \( z'' = \zeta_3 \) will be the same vector \( z' \) found via MVA. Since the \( B_z \) possess a net change at a shock wave its variance will certainly be non-zero and expected to be direction of the largest variance for a CWD (as shown for a shock in Panel E and F of Figure 2).

The normal \( \tau \) to the coplanarity plane (Figure 2) is required to be perpendicular to \( \zeta_3 = z' \) yielding the condition (i) \( \tau \cdot \zeta_3 = 0 \). Since on average \( B \) must reside in the coplanarity plane, we obtain the condition (ii) that \( \tau \cdot < B >= 0 \). As we require a unit \( \tau \) vector, we enforce (iii) \( |\tau|^2 = 1 \). These three conditions determine all components of \( \tau \) to within an unimportant overall sign. The shock normal is then given by \( \mathbf{n} = \tau \times \zeta_3 \), completing a new right handed CVA coordinate system: \( \zeta'_1 = \mathbf{n}; \zeta'_2 = \tau; \zeta'_3 = \zeta_3 \). The matrix of transformation from the original to the CVA coordinate system is given by

\[
L_{(\text{original} \rightarrow \text{HVA})} = \begin{pmatrix}
x & y & z \\
x' & y' & z' \\
\zeta_3' x & \zeta_3' y & \zeta_3' z
\end{pmatrix}
\]

so that the transformed magnetic field would be obtained as \( \mathbf{B}_{\text{HVA}} = L \cdot \mathbf{B}_{\text{original}} \). This matrix implements in one step, a procedure that is essentially two steps in succession:
transforming to MVA followed by a rotation by an angle $\phi^*$ about $\zeta_3$ to achieve the average coplanar condition.

4. Discussion

The geometrical content of the rotation involving $\phi^*$ (implicit in the top two rows of $L$ above) is illustrated in Figure 2. In the background plane (labeled MVA) the orthogonal MVA eigenvector directions $x', y'$ for a shock are indicated. Because of the near degeneracy of the minimum and intermediate eigenvalue’s, these coordinates are not uniquely constrained and could be essentially any pair of perpendicular directions in this plane. In the case of strict eigenvalue degeneracy the actual vectors selected are algorithm dependent! This is generally of now consequence except when a recipe depends on the uniqueness of a direction assigned to one of these two directions as with MVA normals. The choice of $\phi^*$ corresponds to picking two other equally viable perpendicular directions, $x''$ and $y''$ (front plane in Figure 2 labeled CVA) that span the same plane as did $x', y'$ by requiring the $B''$ in CVA to be coplanar on average.

A non-zero choice of $\phi^*$ implies that “the” normal according to CVA, $\zeta'_1$, is no longer the strict minimum variance direction of MVA, $\zeta_1$. In fact, the algebraic construction of $L$ above illustrates that there is no room for an additional minimization of the variance along the normal once the maximum variance direction has been adopted as one of the basis vectors of the CVA system. This also shows that there is no recipe with Lagrange multipliers and minimization that can find the CVA basis, since it is not strictly a direction of minimum variance. There may be a minimization formulation for CVA that asks for the variance along two orthogonal directions $n', r'$ to be maximally equal, with the third basis vector found by the right hand rule. Were such directions found the litmus test of such a technique would be how far
\( n \times \tau \) was from being parallel to the direction of maximum variance that is robustly determined and probably the best vector available for geometry at such layers when only using one vector field.

As shown in the examples in Figure 1 D-F the variance \( Q_1 \) of the field components along the CVA “normal” for shocks remains small because the variance along any direction in the plane labeled MVA/CVA of Figure 2 is small, bounded by the sum of two assumed small eigenvalues: \( \sigma_{n,mva} \leq \lambda_1 + \lambda_2 \). The precondition for attempting the reorientation of the normal by the CVA procedure was that the data set does not determine it very well, and that \( Q_1, Q_2 \) are both “small” and not so far dispersed that \( Q_2 / Q_1 \ll 10 \). (Common sense must enter these decisions, especially when enforcing the eigenvalue ratio tests between eigenvalues that are small and when either value is computed to be below the real noise floor of the measurement.)

In a sense we are inserting the leverage of the existence of the coplanarity plane in place of the poorly determined normal via minimum variance. The maximum variance direction and the coplanarity direction once found in a CWD then determine a better normal (see below for simulations of precision) via the right hand rule.

Conversely, as shown in Figure 1C, when CVA is forced on a RD time series, the result is to put a large \( Q_1 \) on \( \zeta_x \) components in order to make the \( \zeta_y \) components have zero mean; this is not a failure of CVA, but of judgment about the distinctness of the eigenvalue’s of MVA, that implies that they, not CVA, should have been used to determine the geometry.

The basis of the sieve between CVA and MVA then devolves on the judgment of the effective degeneracy of the smallest two eigenvalue’s of the MVA of the time series. Two Figures of Merit, \( \text{FOM}=Q_1, Q_2 \), should be considered. Notice that the FOM of merit goes with
the process and the time series. The success of MVA requires that $Q_2 > 10Q_1$, as is almost satisfied in Panel 1B. Clearly when $FOM = (10^{-4}, 0.4, 0.5999)$ MVA (CVA) does (does not) give a reasonable geometrical reconstruction for the normal, but not a terribly precise idea even of the direction of maximum variance. However, when $FOM = (0.04, 0.1, 0.86)$ CVA (MVA) can (cannot) provide a viable geometry for physical interpretation.

5. Precision of CVA and MVA Geometries at Synthetic Slow and Fast Shocks:

A series of one dimensional magnetosonic shock layers were prepared to contrast the precision of CVA and MVA geometry inversion. The shocks only differ by their compression factor, $\eta = \frac{B_2}{B_1}$, which ranges from 0 to 4, from switch-off slow shocks to high mach number fast mode shock layers. The normal components of shocks of different compression have the same size, leading to a $\Theta_{\beta n}$ variation with $\eta$. Noise localized to the layer is superposed and a realistic net value for $< B_y, >$ is in each profile. (The shock profile in Figure 1D was an example from this set.) MVA and CVA were alternately used on the same time series to reconstruct the geometry and compare it with that known for the simulated profiles of the shocks. Figure 3 summarizes the recovery error in degrees for the normal in the top panel and the coplanarity normal in the bottom panel. The black trace in each panel is the error using MVA; the red trace demonstrates the small error of CVA reconstruction. Except in the vicinity of very weak ($\eta \approx 1$) shocks the CVA process recovered the shock geometry at better than $0.1^\circ$, while MVA rather systematically got the CWD geometry wrong, missing by $25^\circ$. (Errors of this same order were routinely found (Scudder et al 2004a) using MVA on over 100,000 magnetopause traversed based on simulations of 2-D reconnecting current layers.) Since CVA explicitly requires a good
direction of maximum variance, its “failure” at very weak shocks is understandable, since there is little leverage in the transverse fluctuations in that marginally important circumstance.

5.1 Plane Polarized Wave Disturbance: As the first of two brief examples using this technique we consider GGS-Polar magnetometer data on January 31, 2004 at the magnetopause crossing more extensively discussed in two recent papers (Mozer et al, 2004, Scudder et al, 2004b). The upper three panels of Figure 3 are the three components of \( \mathbf{B} \) in the MVA coordinate system determined from data between the vertical dashed lines between approximately 07:46:00 and 07:48:00UT. The MVA procedure has produced \( \mathbf{B}(t) \) with strongest variations along the \( z' \) axis, and less variation along \( x' \) and \( y' \), but both \( x' \) and \( y' \) components have significant average components. The MVA FOM are \( Q_1 = 0.039; \ Q_2 = 0.115 \) with a ratio of 2.9 \( \ll 10 \). Each of the FOM are strongly smaller than \( Q_3 \). Since they are not large relative to one another, “the” minimum variance direction is not well defined.

The small and comparable sizes of the MVA FOM suggest considering CVA for the geometry of this disturbance as a CWD. The lower three panels illustrate the field transformed to the CVA system. The average of the \( y'' \) component of \( \mathbf{B}_{\text{MVA}} \) is zero with only modest variations, while \( B''_x \) is remarkably steady. The CVA FOM are nearly balanced with values \( Q_1 = 0.075; \ Q_2 = 0.079 \). This interval is consistent with being a CWD, a viewpoint not accessible from the MVA representation. The planar wave field is consistent with a linearly polarized large amplitude slow mode wave. The wave is slow mode by showing that the density and magnetic field strength are anti-correlated (Scudder et al, 2004b).
5.2 Candidate for Slow Shock: A second example in the same format is shown in Figure 5 where an abrupt reorientation of the field is seen about 07:45:57UT. The magnetic field transformed to the MVA geometry determined for the interval between the vertical dashed lines is shown in the upper three panels, while the MVA geometry inversion is illustrated in the lower three panels. The MVA FOM are \(Q_1 = 0.049; Q_2 = 0.184\), small and close together with ratio \(3.7 << 10\). The magnetic field organized in MVA has a strong average \(y'\) component. In keeping with our arguments above, the MVA geometry is not warranted. The CVA approach finds a candidate coplanarity plane; the CVA FOM of \(Q_1 = 0.126; Q_2 = 0.107\) are nearly balanced as we saw in Panel 1F in our simulations with a CVA recovery of a CWD. The physical interpretation in the MVA system is confused at best, while in CVA this interval is consistent with a CWD with a relatively sharp field magnitude change within it. Of particular interest is the planar structure with a nearly complete “switching-off” of the transverse \(B_z\) component, reminiscent of the behavior of the strongest (switch-off) slow shocks. The abrupt change and the associated density increase (not shown) make this CWD a candidate for a slow shock, pending further R-H testing.

A successful \((Q_1 \approx Q_2)\) CVA geometry does not prove that the time series is a shock, but provides a test that it is consistent with being a planar vector field. A possible reason for this planar condition (especially in the presence of the abrupt change in the \(z''\) component) is that this is a shock layer, although that conclusion and its type (fast or slow) is not implied until further Rankine-Hugoniot conservation laws beyond coplanarity are verified as are done for this example in Scudder et al (2004b).

6. Summary
CVA performs well with high precision on simulated and real data and should actively be considered for geometrical reconstructions in tandem with MVA techniques. In the framework of hypothesis testing, MVA is best adapted to answering the question “Is it a 1-D torsional shear layer”, while CVA is best adapted to answering the question “Is it a 1-D coplanar shear wave”. Together they can be used to test these two hypotheses, and failing both also provide a categorization system for “other” shear layers of mixed or multidimensional character that should not be interpreted as if they were one dimensional transects. In this way physical models can be evaluated in the best geometries that can be determined from the limited world line data that satellites provide.

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Figure Captions:

Figure 1. Left column (Panel A-C) pertain to simulated RD. Right column (Panel D-F) pertain to simulated Fast Shock. All panels: Thick curves depict the variation of three Cartesian components of $B$ across time series. Light blue (x); two components transverse to x are illustrated in red (y) and green (z). Temporal average of each component is given by dashed line of the same color. Variance about the mean of each component is indicated by vertical error bar of the same color, anchored with a diamond on the average dotted line. $\lambda_{1}, \lambda_{2}$ indicate $(\lambda_{1}, \lambda_{2})/(\lambda_{1} + \lambda_{2} + \lambda_{3})$. **Panel (a):** RD with x direction along the normal. **Panel (b):** Same RD as in Panel A described in MVA geometry: x along the minimum variance; z direction of maximum variance. **Panel (c):** Same RD as in Panel A described in CVA geometry: y perpendicular to coplanarity plane, z in direction of maximum variance of MVA. **Panel (d):** Fast Shock in laboratory coordinates. **Panel (e):** Same Fast Shock as in Panel 1D but viewed in MVA geometry: x direction of minimum variance, z direction of maximum variance. **Panel (f):** Same Fast shock as in 1(d) in CVA geometry: y coplanarity plane normal, z maximum variance direction.

Figure 2. Isometric drawing of the geometry of magnetic tubes of force at a slow shock layer (S) for the purpose of illustrating the issues with MVA coordinate system ($x', y', z'$) and the newly proposed CVA coordinate system ($x'', y'', z''$) for time series where it is suspected that the vector field is coplanar, as in a shock. The normal $n$ to the shock and to the coplanarity plane $\tau$ are indicated. The equivalent planes (MVA), (CVA) spanned by eigenvector’s of potentially degenerate eigenvalue’s are also indicated. The split coplanarity plane is labeled C and C’.

Figure 3. Inventory of the precision of recovery of CWD geometry at slow and fast mode shocks as a function of compression ratio $\eta = \frac{B_{x}}{B_{i}}$, using MVA (black) and CVA (red). Upper (lower) panel depicts the error in the normal (coplanarity normal) directions from the model.

Figure 4. Polar data for January 31, 2004 contrasting the magnetometer data at the earth’s magnetopause in MVA vs CVA coordinate representation, illustrating the detection of a linearly polarized slow mode wave.

Figure 5. Polar data for January 31, 2004 contrasting the magnetometer data at the earth’s magnetopause in MVA vs CVA coordinate representation, illustrating the detection of a candidate for a slow shock wave.
References:


Mozer, F.S., S.D. Bale and J.D. Scudder (June 2004), Large Amplitude, Extremely Rapid, Predominantly Perpendicular Electric Field Structures at the Magnetopause, accepted, GRL.


Figure 1. Left column (Panels a-c) pertain to simulated RD. Right column (Panel d-f) pertain to simulated Fast Shock. All panels: Thick curves depict the variation of three Cartesian components of $B$ across time series. Light blue (x); two components transverse to x are illustrated in red (y) and green (z). Temporal average of each component is given by dashed line of the same color. Variance about the mean of each component is indicated by vertical error bar of the same color, anchored with a diamond on the average dotted line. FOM(1,2) indicate $(\lambda_1, \lambda_2)/(\lambda_1 + \lambda_2 + \lambda_3)$. Panel (a): RD with x direction along the normal. Panel (b): Same RD as in Panel A described in MVA geometry: x along the minimum variance; z direction of maximum variance. Panel (c): Same RD as in Panel A described in CVA geometry: y perpendicular to coplanarity plane, z in direction of maximum variance of MVA. Panel (d): Fast Shock in laboratory coordinates. Panel (e): Same Fast Shock as in Panel 1(d) but viewed in MVA geometry: x direction of minimum variance, z direction of maximum variance. Panel (f): Same Fast shock as in 1(d) in CVA geometry: y coplanarity plane normal, z maximum variance direction.
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